AFRICAN ECONOMIC RESEARCH CONSORTIUM

Collaborative MA Programme in Economics for Anglophone Africa
(Except Nigeria)

JOINT FACILITY ELECTIVES
June – October 2004

ECONOMETRICS THEORY & PRACTICE

Second Session: Final Examination

Time: 9.30am – 12.30pm | Tuesday September 28, 2004

INSTRUCTIONS: This paper consists of FIVE questions. You are required to answer any FOUR questions.

Question 1

(a) Find the mean, variance and autocovariances for each of the following stochastic processes: **(5 marks**

$$\text{MA}(1): Y_t = \mu + \varepsilon_t + \theta\varepsilon_{t-1}$$

$$\text{AR}(1): Y_t = \mu + \varphi Y_{t-1} + \varepsilon_t$$

Where ε_t is a white noise process with mean zero and constant variance, σ^2.

(b) Explain the main steps in Box-Jenkins modelling. **(3 marks)**

(c) Explain how you would carry out cointegration test using the Engle-Granger procedure. What are the main defects of the procedure? **(7 marks)**

Question 2

(a) Distinguish between a pure random walk process and a random walk with drift process. **(2 marks)**

(b) Using your own notation, find the mean, variance, and first and second autocovariances of a pure random walk, and comment on stationarity. **(4 points)**

(c) Using your own notation, show that the first difference of a process characterized by a random walk with drift is stationary. **(4 points)**

(d) Discuss the contention that cointegration test can be perceived as a unit root test within the multivariate time series framework. **(5 marks)**

Question 3

(a) You are given two I(1) series, x_t and y_t, which can be decomposed into stochastic trend and irregular components as follows

$$x_t = \mu_{xt} + \varepsilon_{xt}$$

$$y_t = \mu_{yt} + \varepsilon_{yt}$$

where ε_{it} $(i = x, y)$ is a stationary random variable, and stochastic trends (random walks) are specified as follows:

$$\mu_{xt} = \mu_{xt-1} + \eta_{xt}$$

$$\mu_{yt} = \mu_{yt-1} + \eta_{yt}$$

Show that x_t and y_t must have the same stochastic trend up to the scalar $-\beta_2 / \beta_1$, if there exist non-zero values β_1 and β_2 for which the linear combination $\beta_1 x_t + \beta_2 y_t$ is stationary (i.e. x_t and y_t are cointegrated). **(9 points)**

(b) Which of the following stochastic vector processes (VAR models) are covariance stationary? **(6 points)**

Model 1

$$\mathbf{y}_t = \mathbf{v} + \Theta \mathbf{y}_{t-1} + \varepsilon_t$$

where $\mathbf{y}_t = (y_{1t}, y_{2t})'$, $\mathbf{v} = (v_1, v_2)'$ is a vector of intercept terms, $\Theta = \begin{bmatrix} 0.108 & 0.461 \\ 0.43 & 0.207 \end{bmatrix}$ is a (2×2) matrix of fixed coefficients, and $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})$ is a 2-dimensional *white noise* or *innovation process*, that is $E(\varepsilon_t) = 0$, $E(\varepsilon_t, \varepsilon_t') = \Sigma_\varepsilon$, and $E(\varepsilon_t, \varepsilon_s') = 0$ for $s \neq t$. Σ_ε is the covariance matrix.

Model 2

$$\mathbf{y}_t = \mathbf{v} + A\mathbf{y}_{t-1} + u_t$$

where $\mathbf{y}_t = (y_{1t}, y_{2t})'$, $\mathbf{v} = (v_1, v_2)'$ is a vector of intercept terms, $A = \begin{bmatrix} 0.78 & 0.26 \\ 0.001 & 0.67 \end{bmatrix}$ is a (2×2) matrix of fixed coefficients, and $u_t = (u_{1t}, u_{2t})$ is a 2-dimensional *white noise* or *innovation process*, that is $E(u_t) = 0$, $E(u_t, u_t') = \Sigma_u$, and $E(u_t, u_s') = 0$ for $s \neq t$. Σ_u is the covariance matrix.

Model 3

$$y_{1t} = 0.5y_{1t-1} + 0.8y_{2t-1} + \varepsilon_{1t}$$
$$y_{2t} = 0.9y_{1t-1} + 0.02y_{2t-1} + \varepsilon_{2t}$$

where $E(\varepsilon_{1t}\varepsilon_{1\tau}) = 1$ for $t = \tau$ and 0 otherwise, $E(\varepsilon_{2t}\varepsilon_{2\tau}) = 2$ for $t = \tau$ and 0 otherwise, and $E(\varepsilon_{1t}\varepsilon_{2\tau}) = 0$ for all t and τ.

Question 4

(a) Suppose you have a two-period panel data, and you have the following unobserved effects model:

$$y_{it} = \beta_0 + \lambda_0 d2_t + \beta_1 x_{it} + \alpha_i + \varepsilon_{it},\ \ t = 1,2. ----(1)$$

$$\alpha_i \sim N(0, \sigma_\alpha^{\ 2})$$

$$\varepsilon_{it} \sim N(0, \sigma_\varepsilon^2)$$

$$E(\alpha_i \varepsilon_{it}) = 0 \quad E(\alpha_i \alpha_j) = 0 \ (i \neq j)$$

$$E(\varepsilon_{it}\varepsilon_{is}) = E(\varepsilon_{it}\varepsilon_{jt}) = E(\varepsilon_{it}\varepsilon_{js}) = 0 \ (i \neq j; t \neq s)$$

where i denotes a cross section unit (it can be a firm, or a household), t denotes time period. $d2_t$ is a dummy variable. It is equal to zero when $t = 1$, and equals to one when $t = 2$. The dummy does not change across i. α_i captures all unobserved, time invariant factors that influence y_{it}. ε_{it} is the idiosyncratic error, representing factors that change over time, and affect y_{it}.

(i) Show that even when the unobserved, time invariant factors represented by α_i are not correlated with the explanatory variable, x_{it}, the pooled OLS estimator is not appropriate for the estimation of equation (1). Which method would you use to estimate the equation? (6 points)

(ii) Derive the fixed effects estimator. (State the major assumption(s) that would justify the use of the fixed effects estimator). (6 points)

(b) Given the following panel data model

$$y_{it} = \beta_0 + \beta_1 x_{it1} + \ldots\ldots + \beta_k x_{itk} + \alpha_i + \varepsilon_{it} \ldots\ldots(1)$$

$$i = 1,\ldots\ldots,N;\ t = 1,\ldots\ldots,T$$

$$\alpha_i \sim N(0, \sigma_\alpha^{\ 2})$$

$$\varepsilon_{it} \sim N(0, \sigma_\varepsilon^2)$$

$$E(\alpha_i \varepsilon_{it}) = 0 \quad E(\alpha_i \alpha_j) = 0 \ (i \neq j)$$

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Suppose you decide to estimate the model using the random effects method using the following GLS estimator:

$$y_{it} - \lambda\bar{y}_i = \beta_0(1-\lambda) + \beta_1(x_{it1} - \lambda\bar{x}_{i1}) + \dots\dots$$
$$+ \beta_k(x_{itk} - \lambda\bar{x}_{ik}) + (v_{it} - \lambda\bar{v}_i)\dots..(2)$$

where $v_{it} = \alpha_i + \varepsilon_{it,}$ and $\lambda = 1 - [\sigma_\varepsilon^2 / (\sigma_\varepsilon^2 + T\sigma_\alpha^2)]^{1/2}$

Under which condition(s) are the random effects (RE) estimates close to pooled OLS estimates?; and when are the RE estimates close to fixed effects (FE) estimates? (3 points)

Question 5

(a) When one or more explanatory variables in a regression model are binary, we can represent them as dummy variables, and estimate a linear regression model. However, the application of the linear regression model when the *dependent variable* is binary is more complex. Discuss. (6 points)

(b) Suppose you want to estimate a model in which a binary variable, a decision to participate in the labor force (Y), is determined by a continuous variable, the wage rate (X); and you decide to adopt the following representation:

$$P_i = E(Y = 1 \mid X_i) = \frac{1}{1 + e^{-(\alpha + \beta X_i)}}$$

where X is wage rate, Y=1 means individual i partipates in the labor force, Y=0 means an individual does not participate in the labor market.

Show that unlike the linear probability model, this representation will not produce "nonsensical" predictions, i.e. predictions outside the $(0 \le P_i \le 1)$ range. (4 points)

(c) Given the simplest version of the tobit model

$$y_i^c = \boldsymbol{X}_i\beta + u_i, \quad u_i \sim NID(0,\sigma^2)$$

$$y_i = y_i^c \text{ if } y_i^c > 0; \; y_i = 0 \text{ otherwise}$$

where y_i^c is a latent variable that is observed whenever it is positive. When the latent variable is negative, the observation is censored.

Derive the loglikelihood function for the (tobit) model, and comment on the appearance (form) of the loglikelihhod function. (5 points) where i denotes a cross section unit (it can be a firm, or a household), t denotes time period. α_i captures all unobserved, time invariant factors that influence y_{it}. ε_{it} is the idiosyncratic error, representing factors that change over time, and affect y_{it}.

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